September 8th, 2015 **Name (Please Print)** _____
Probability **Midterm** **Semester I 2015/16**

1.(15 points) Suppose we toss a fair coin two times. Let X denote the number of heads on the first toss, Y denote the number of tails on the first toss and Z denote the number of heads on the second toss.

(a) Write out the Sample space and define the Probability function for the above experiment.

(b) Show that $X, Y, Z \sim$ Bernoulli($\frac{1}{2}$).

(c) Let $W = X + Y$. Find the distribution of W.

(d) Let $U = X + Z$. Find the distribution of U.

(e) Do W and U have the same distribution ? Explain your answer.

2. (15 points)Shyam is randomly selected from the citizens of Hyderabad by the Health authorities. A laboratory test on his blood sample tells Shyam that he has tested positive for H1N1 virus. It is found that 95% of people with H1N1 test positive but 2% of people without the disease will also test positive. Suppose that 1% of the population has the disease. What is the probability that Shyam does not have H1N1 disease ?

3. (15 points) Let $0 < p, q < 1$. Let X be a Geometric (p) random variable and Y be an independent Geometric(q) random variable. Let $W = \max\{X, Y\}$. Find the distribution of W.

4. (15 points) Let X and Y be discrete random variables with Range (X) $=\{0, 1, 2\}$ and Range (Y) $= \{1, 2\}$ with joint distribution given by the chart below.

	$X = 0$	$X = 1$	$X = 2$
$Y = 1$	0.1	0.2	0.1
$Y = 2$	0.3	0.2	0.1

(a) What is the marginal probability mass function of Y ?

(b) What is the value of $P(X = 1 \mid Y = 2)$?

(c) Find $E[XY]$.

(d) Find $\text{Cov}(X, Y) := E[XY] - E[X]E[Y]$.

(e) Are X and Y independent ?